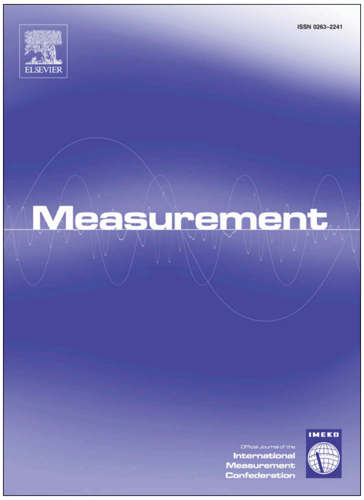

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Muhammad Aslam, Rashad A. R. Bantan, Nasrullah Khan

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# Design of Tests for Mean and Variance under Complexity-An Application to Rock Measurement Data

[1*]Muhammad Aslam, [2]Rashad A. R. Bantan, [3]Nasrullah Khan

[1]Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia; Email: aslam_ravian@hotmail.com

[2]Department of Marine Geology, Faculty of Marine Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia; Email: rbantan@kau.edu.sa

[3]Department of Statistics, UVAS, Lahore, Jhang campus. Email: nas_shan1@hotmail.com

1. existing t-test under classical statistics cannot be applied when the data is uncertain
2. t-tests under the neutrosophic statistics
3. proposed tests for testing the means of single and two neutrosophic populations
4. comparison of the proposed tests with the t-tests under classical statistics

## Abstract

The student t-test and F-test are applied for testing the hypothesis of the population mean and variance, respectively. The existing t-test and F-test under classical statistics cannot be applied when the data is uncertain, indeterminate, and fuzzy. Neutrosophic statistics is applied when data is indeterminate, uncertain, and vagueness. This paper introduces the t-test and F-test under neutrosophic statistics. The designing of the proposed tests for testing the population means and testing the equality of population variances are presented. The comparison of the proposed tests with the t-test and F-test under classical statistics is also given. A real example from rock engineering is taken for the illustration of the proposed tests. The comparative study shows that the proposed tests are effective, flexible, and informative to be applied in uncertain environment.

**Keywords:** t-test; F-test; sample; Neutrosophy; population; statistics

*corresponding author

## 1 Introduction

The statistical tests are playing an important role in decision making in a variety of fields. Among these statistical tests, the F-test and t-test are applied for the testing of the equality of the population variances and population means. Statistical tests that are applied to compare the means and variances of two populations are called t-test and F-test, respectively. The t-test works when the data is small (when the sample size is less than 30) and follows the normal distribution. On the other hand, in F-test, the test statistic has an F-distribution and applied when the sample is at least 30. Both statistical tests have a limitation that both tests are applied only when the sample is selected from the normal distribution. In addition, the t-test is applied when the standard deviation is unknown in practice. This testing is carried based on sample information. The null hypothesis of equal population variances and means is accepted if the sample information support it, otherwise the null hypothesis is rejected in favor of an alternative hypothesis and concluded that both the population does not have the same variances and means. Due to the many applications of the F-test and t-test, several authors worked on them in a variety of ways. Ref. [1] discussed the overlapping in confidence interval (C.I) of the t-test for two pupations. Ref. [2] applied the t-test to see the significant factor for human immunodeficiency virus (HIV) patients. Ref. [3] applied the statistical tests in the healthcare department. Ref. [4] presented the C.I when the ratio of variances is known.

The tests under classical statistics are applied under the assumption that all observations in a data set are determined, precise, and certain. According to [5] “statistical data are frequently not precise numbers but more or less non-precise, also called fuzzy. Measurements of continuous variables are always fuzzy to a certain degree”. Therefore, when the data in hand is fuzzy, the traditional tests under classical statistics are not suitable to apply to this type of data. Therefore, the test under the fuzzy logic is an alternative to test under classical statistics. Ref. [6] discussed the fuzzy p-value for testing of hypothesis. Ref. [7] discussed the fuzzy paired t-test for two normal populations. Ref. [8] proposed the test for testing the fuzzy hypothesis for equality of mean. Ref. [9] presented the statistical tests using the fuzzy approach. Ref. [10] used the fuzzy confidence interval for testing of hypothesis. Ref. [11] applied a fuzzy test in the education department. Ref. [12] proposed the fuzzy t-test for trapezoidal fuzzy numbers. More applications can be seen in [13] and [14].

Ref. [15] introduced the neutrosophic logic as the extension of the fuzzy logic. The fuzzy logic has two degrees of measures such as true and false. The neutrosophic fuzzy logic consists of three measures such as true, false, and indeterminacy. In addition, [16] discussed that neutrosophic logic is also more efficient than methods used to analyze the interval data. More details on neutrosophic logic and applications can be seen in [17], [18], [19] [20] and [21]. Ref. [22] introduced the descriptive statistics using the neutrosophic logic is called neutrosophic statistics (NS). The NS is an extension of classical statistics and applied for the analysis of the imprecise and indeterminate data. [23] and [24] introduced the neutrosophic numbers and discussed the application for rock engineering. [25] introduced the NS in the area of statistical control the first time. Ref. [26] introduced analysis of variance test using the NS. Ref. [27] discussed the application of NS for prostate cancer. Ref. [28] presented the coefficient of variation using the neutrosophic statistics.

Ref. [22] introduced the t-test for NS. But, as pointed by [24] that [22] method is not simple for engineering applications. In this paper, the proposed F-test and t-test under NS using [24] method are given. The proposed tests will be discussed with the help of real data of rock joint roughness coefficient (JRC). It is expected that the proposed tests will be more effective than the tests under classical statistics.

## 2 Design of the Proposed Tests

In this section, the designing of the proposed t-test and F-test under NS is presented. The neutrosophic F-test will be applied for the testing of equality of two neutrosophic population variances and neutrosophic t-test will be applied for testing of the neutrosophic population mean. Both tests are designed under the assumption that the samples are drawn from an incomplete, fuzzy, and indeterminate normal population. Further, the neutrosophic samples are independent.

### *2.1 t- test for Single Neutrosophic Population*

Consider a neutrosophic random sample $n_N \epsilon [n_L, n_U]$ from a neutrosophic normal independent population with neutrosophic mean, say $\mu_N \epsilon [\mu_L, \mu_U]$ and variance $\sigma_N^2 \epsilon [\sigma_L^2, \sigma_U^2]$. Let $z_N = a_N + b_N I_N; I_N \epsilon [I_L, I_U]$ be a neutrosophic random variable, where $a_N$ and $b_N I_N$ are determinate and indeterminate parts, respectively and $I_N \epsilon [I_L, I_U]$ denotes the indeterminacy interval. By following [23] and [24], note here that when $I_L = 0$, $z_N \epsilon [z_L, z_U]$ reduces to the random variable under CS. Let assume here that $\mu_N \epsilon [\mu_L, \mu_U]$ and $\sigma_N^2 \epsilon [\sigma_L^2, \sigma_U^2]$ are unknown and $n_N \epsilon [n_L, n_U]$ is less than 30. Let $\mu_{0N}$ be a specified value. The proposed t-test will be applied to test the null hypothesis $H_{0N}$: $\mu_N = \mu_{0N}$ vs. the alternative hypothesis $H_{1N}$: $\mu_N \neq \mu_{0N}$.

From [23], the mean and variance of $z_N \epsilon [z_L, z_U]$ are given as

$$\bar{z}_N = \bar{a}_N + \bar{b}_N I_N \tag{1}$$

$$s_N^2 = \frac{1}{n_N} \sum_{i=1}^{n_N} \left( z_{n_N} - \bar{z}_N \right)^2 \tag{2}$$

where $\bar{a}_N$ and $\bar{b}_N$ are the averages of determined and indeterminate parts, respectively and $\left( z_{n_N} - \bar{z}_N \right)^2$ is given by

$$\left( z_{n_N} - \bar{z}_N \right)^2 = \begin{bmatrix} \min \begin{pmatrix} (a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U) \\ (a_i + b_i I_U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U) \end{pmatrix} \\ \max \begin{pmatrix} (a_i + b_i I_L)(\bar{a} + \bar{b} I_L), (a_i + b_i I_L)(\bar{a} + \bar{b} I_U) \\ (a_i + b_i I^U)(\bar{a} + \bar{b} I_L), (a_i + b_i I_U)(\bar{a} + \bar{b} I_U) \end{pmatrix} \end{bmatrix}, I_N \epsilon [I_L, I_U] \tag{3}$$

The neutrosophic t-test for the single population is given as follows

$$t_N = \frac{[\bar{a}_N + \bar{b}_N I_N] - \mu_N}{\frac{1}{n_N} \sum_{i=1}^{n} \left( z_{n_N} - \bar{z}_N \right)^2} ; \mu_N \epsilon [\mu_L, \mu_U] \tag{4}$$

Note here that the proposed neutrosophic t-test reduces to t-test under classical statistics if no indeterminacy is found in the observations from the complex system.

The proposed test statistic has a degree of freedom say, $\vartheta_N = n_N - 1$, the null hypothesis $H_{0N}: \mu_N = \mu_{0N}$ is rejecting if $t_N \epsilon [t_L, t_U]$ is greater than the tabulated value at the level of significance $\alpha$. The operational procedure of the proposed t-test is shown in Figure 1. The $(1-\alpha) \times 100\%$ neutrosophic confidence interval (N.C.I) is given by

$$\left\{(\bar{a}_N + \bar{b}_N I_N) - t_{N,\vartheta_N,\alpha}\frac{s_N}{\sqrt{n_N}}, (\bar{a}_N + \bar{b}_N I_N) + t_{N,\vartheta_N,\alpha}\frac{s_N}{\sqrt{n_N}}\right\} \tag{5}$$

The neutrosophic standard deviation is defined as

$$s_N = \sqrt{\frac{1}{n_N}\sum_{i=1}^{n}\left(z_{n_N} - \bar{z}_N\right)^2} \tag{6}$$

### *2.2 F-test for Testing Two Neutrosophic Variances*

The F-test is applied for testing the hypothesis that two neutrosophic populations have equal neutrosophic variances. The F-test has been widely used for testing the means of more than two populations in the analysis of variance, see [26]. Like the t-tests, the F-test is also applied under the assumption that the sample should be drawn from the neutrosophic population and must be independent. The F-test is used to test the null hypothesis that two variances are equal versus the alternative hypothesis that the variances are not equal. Suppose that $\sigma_{N1}^2 \epsilon [\sigma_{L1}^2, \sigma_{U1}^2]$ and $\sigma_{N2}^2 \epsilon [\sigma_{L2}^2, \sigma_{U2}^2]$ are two neutrosophic population variances and assumed to the unknown in practice. Let $s_{N1}^2 \epsilon [s_{L1}^2, s_{U1}^2]$ and $s_{N2}^2 \epsilon [s_{L2}^2, s_{U2}^2]$ denote the sample variances from the first and second populations, respectably. Suppose also that $n_{N1} \epsilon [n_{L1}, n_{U1}]$ and $n_{N2} \epsilon [n_{L2}, n_{U2}]$ be the sample size for the first and second populations, respectably. Based on the information, the proposed F-test will be applied to test the null hypothesis $H_{0N}: \sigma_{N1}^2 = \sigma_{N2}^2$ vs. the alternative hypothesis that $H_{1N}: \sigma_{N1}^2 \neq \sigma_{N2}^2$.

The F-test under the neutrosophic statistics by assuming the equal population variance is given by

$$F_N = \frac{s_{N1}^2}{s_{N2}^2} \tag{7}$$

where $s_{N1}^2$ and $s_{N2}^2$ are given as

$$s_{N1}^2 = \frac{1}{n_{N1}}\sum_{i=1}^{n_{N1}}\left(z_{n_{N1}} - \bar{z}_{N1}\right)^2 \tag{8}$$

$$s_{N2}^{2}=\frac{1}{n_{N2}}\sum_{i=1}^{n_{N2}}\left(z_{n_{N2}}-\bar{z}_{N2}\right)^{2} \qquad (9)$$

Note here that the proposed neutrosophic F-test reduces to F-test under classical statistics if no indeterminacy is found in the observations from the complex system. Note here also that $\left(z_{n_{N1}}-\bar{z}_{N1}\right)^{2}$ and $\left(z_{n_{N2}}-\bar{z}_{N2}\right)^{2}$will be given as follows

$$\left(z_{n_{N1}}-\bar{z}_{N1}\right)^{2}=\begin{bmatrix}\min\begin{pmatrix}(a_{i1}+b_{i1}I_L)(\bar{a}_1+\bar{b}_1I_L),(a_{i1}+b_{i1}I_L)(\bar{a}_1+\bar{b}_1I_U)\\(a_{i1}+b_{i1}I_U)(\bar{a}_1+\bar{b}_1I_L),(a_{i1}+b_{i1}I_U)(\bar{a}_1+\bar{b}_1I_U)\end{pmatrix}\\ \max\begin{pmatrix}(a_{i1}+b_{i1}I_L)(\bar{a}_1+\bar{b}_1I_L),(a_{i1}+b_{i1}I_L)(\bar{a}_1+\bar{b}_1I_U)\\(a_{i1}+b_{i1}I_U)(\bar{a}_1+\bar{b}_1I_L),(a_{i1}+b_{i1}I_U)(\bar{a}_1+\bar{b}_1I_U)\end{pmatrix}\end{bmatrix}, I_N\epsilon[I_L,I_U]$$

(10)

$$\left(z_{n_{N2}}-\bar{z}_{N2}\right)^{2}=\begin{bmatrix}\min\begin{pmatrix}(a_{i2}+b_{i2}I_L)(\bar{a}_2+\bar{b}_2I_L),(a_{i2}+b_{i2}I_L)(\bar{a}_2+\bar{b}_2I_U)\\(a_{i2}+b_{i2}I_U)(\bar{a}_2+\bar{b}_2I_L),(a_{i2}+b_{i2}I_U)(\bar{a}_2+\bar{b}_2I_U)\end{pmatrix}\\ \max\begin{pmatrix}(a_{i2}+b_{i2}I_L)(\bar{a}_2+\bar{b}_2I_L),(a_{i2}+b_{i2}I_L)(\bar{a}_2+\bar{b}_2I_U)\\(a_{i2}+b_{i2}I_U)(\bar{a}_2+\bar{b}_2I_L),(a_{i2}+b_{i2}I_U)(\bar{a}_2+\bar{b}_2I_U)\end{pmatrix}\end{bmatrix}, I_N\epsilon[I_L,I_U]$$

(11)

The proposed F-test has neutrosophic F-distribution with $n_{N1}-1$ degree of freedom for the first sample and $n_{N2}-1$ degree of freedom for the second sample. The null hypothesis of equality of variances is rejected if $F_N\epsilon[F_L,F_U]$ is greater than the critical value at a specified level of significance. The operational procedure of the proposed F-test is shown in Figure 2.

### *2.3 t-test for Two Neutrosophic Populations*

The designing of the neutrosophic t-test for neutrosophic populations having equal neutrosophic variances is presented. The proposed neutrosophic t-test will be applied only if the null hypothesis of equal variances is accepted by the neutrosophic F-test in Section 2.2. For the application of this test, it is assumed that the neutrosophic samples are independent and drawn from the neutrosophic population having equal variances. The null hypothesis that means are equal versus the alternative hypothesis that means are not equal are tested using the following statistic

$$t_N = \frac{(\bar{z}_{N1} - \bar{z}_{N2}) - d_{0N}}{s_{pN}\sqrt{\frac{1}{n_{N1}} + \frac{1}{n_{N2}}}}\ ; n_{N1}\epsilon[n_{L1},n_{U1}],\ n_{N2}\epsilon[n_{L2},n_{U2}] \qquad (12)$$

where $d_{0N}$ denotes the difference between means of neutrosophic population.

where $s_{pN}$ is called the neutrosophic pooled variance and defined as follows

$$s_{pN} = \frac{(n_{N1} - 1)s_{N1}^2 + (n_{N2} - 1)s_{N2}^2}{n_{N1} + n_{N2} - 2} \qquad (13)$$

The null hypothesis of equal neutrosophic means is rejecting if the calculated value of the statistic $t_N\epsilon[t_L,t_U]$ is greater than the tabulated value at the level of significance $\alpha$ and degree of freedom $n_{N1} + n_{N2} - 2$.

## 3 Applications of the Proposed Tests

In this section, the application of the proposed t-tests for the two-neutrosophic population using real rock JRC data is discussed. According to [23] "the original roughness profiles were measured by using profilograph and a roughness ruler on a natural rock joint surface in Changshan County of Zhejiang Province, China". The neutrosophic data for measurement orientation $\theta$ and sample length $L$ is shown in Table 1. More details about the data can be seen in [29]. The rock engineer is interested to know whether the population means of lengths at 10cm and 20cm are equal or not.

**Table 1:** The rock JRC data [23]

| $L$ | 10cm | | 20cm | |
|---|---|---|---|---|
| $\theta$ | $a_{i1}$ | $b_{i1}$ | $a_{i1}$ | $b_{i1}$ |
| $0^o$ | 8.3040 | 4.4771 | 7.9370 | 3.4325 |
| $15^o$ | 8.4719 | 4.4784 | 8.2300 | 3.4759 |
| $30^o$ | 8.2415 | 4.7057 | 7.9003 | 4.0572 |
| $45^o$ | 7.6124 | 4.6240 | 7.4768 | 3.4474 |
| $60^o$ | 6.5660 | 4.9185 | 6.4234 | 3.9627 |
| $75^o$ | 5.8289 | 4.2126 | 5.6847 | 3.5513 |
| $90^o$ | 4.6413 | 4.8108 | 4.8432 | 3.6965 |
| $105^o$ | 5.3661 | 4.821 | 5.4661 | 3.5119 |
| $120^o$ | 6.8074 | 4.6500 | 6.6243 | 3.7926 |
| $135^o$ | 7.3153 | 3.8208 | 7.0258 | 3.0824 |
| $150^o$ | 8.0308 | 4.8731 | 7.6585 | 3.8130 |
| $165^o$ | 8.3945 | 4.4180 | 8.3040 | 3.3213 |
| $180^o$ | 7.7062 | 4.2877 | 7.2427 | 3.7113 |
| $195^o$ | 7.7130 | 4.4507 | 7.3968 | 3.6661 |

| | | | | |
|---|---|---|---|---|
| $210^{o}$ | 7.6459 | 3.8887 | 7.4116 | 3.0596 |
| $225^{o}$ | 6.9402 | 3.9529 | 6.8294 | 2.8512 |
| $240^{o}$ | 6.0125 | 3.6913 | 5.8648 | 2.8769 |
| $255^{o}$ | 5.2825 | 3.8683 | 5.4738 | 2.7801 |
| $270^{o}$ | 4.6859 | 4.2330 | 4.6748 | 3.2749 |
| $285^{o}$ | 5.4587 | 3.0948 | 5.3757 | 2.2367 |
| $300^{o}$ | 6.7272 | 3.4897 | 6.4594 | 2.7061 |
| $315^{o}$ | 7.6638 | 4.9579 | 7.3405 | 4.2297 |
| $330^{o}$ | 7.5450 | 4.5689 | 7.3872 | 3.2277 |
| $345^{0}$ | 7.6639 | 4.0383 | 7.5456 | 3.2610 |

Let length at 10cm denotes sample 1 and lengths at 20cm presents sample 2. The neutrosophic null hypothesis $H_{0N}{:}\mu_{1N}=\mu_{2N}$ will be tested against the alternative hypothesis $H_{0N}{:}\mu_{1N}\neq\mu_{2N}$. Before the testing of equality of means, the rock engineer should perform the neutrosophic F-test to see whether the variances of two populations are equal or not. Therefore, some necessary calculations by following [23] for the proposed neutrosophic t-test and F-test are given as follows

The neutrosophic arithmetic mean of sample 1 and sample 2 are computed as follows

$$\bar{a}_1=\frac{1}{24}\sum_{i=1}^{24}a_{i1}=\frac{8.304+\ldots+7.6639}{24}=6.9427$$

$$\bar{a}_2=\frac{1}{24}\sum_{i=1}^{24}a_{i2}=\frac{7.9370+\ldots+7.5456}{24}=6.7740$$

The indeterminate coefficient of sample 1 and sample 2 are computed as follows

$$\bar{b}_1=\frac{1}{24}\sum_{i=1}^{24}b_{i1}=\frac{4.4771+\ldots+4.0383}{24}=4.3055$$

$$\bar{b}_2=\frac{1}{24}\sum_{i=1}^{24}b_{i2}=\frac{3.4325+\ldots+3.2610}{24}=3.3761$$

The neutrosophic average values of sample 1 and sample 2 are given by

$$\bar{z}_{N1}=6.9427+4.3055I_N;\ I_N\epsilon[0,1]$$

$$\bar{z}_{N2}=6.7740+3.3761I_N;\ I_N\epsilon[0,1]$$

The square of differences for sample 1 and sample 2, $Z_{i1}(i=1,2,\ldots,24)$ are given as follows

$$(z_{11}-\bar{Z}_1)^2=\begin{bmatrix}\min\left((a_{11}-\bar{a}_1)^2,\left((a_{11}-\bar{a}_1)\left((a_{11}-\bar{a}_1)+1\times(b_{11}-\bar{b}_1)\right),(a_{11}-\bar{a}_1)+1\times(b_{11}-\bar{b}_1)^2\right)\right)\\ \max\left((a_{11}-\bar{a}_1)^2,\left((a_{11}-\bar{a}_1)\left((a_{11}-\bar{a}_1)+1\times(b_{11}-\bar{b}_1)\right),(a_{11}-\bar{a}_1)+1\times(b_{11}-\bar{b}_1)^2\right)\right)\end{bmatrix},$$

$I_N\epsilon[0,1]$

$(z_{11} - \overline{Z}_1)^2 = [1.8530, 2.3495] \ldots$ [0.2061, 1.4375] $I\epsilon[0,1]$ and similarly for sample 2. The corresponding neutrosophic variance for sample 1 and sample 2 are given as follows

$$s_{N1}^2 = \frac{1}{23}\sum_{i=1}^{24}\left(z_{n_{N1}} - \bar{z}_{N1}\right)^2 = [1.1807, 2.0664]$$

$$s_{N2}^2 = \frac{1}{23}\sum_{i=1}^{24}\left(z_{n_{N2}} - \bar{z}_{N2}\right)^2 = [0.9789, 1.7360]$$

The values of the neutrosophic F-test is calculated as follows

$$F_N = \frac{s_{N1}^2}{s_{N2}^2} = \frac{[1.1807, 2.0664]}{[0.9789, 1.7360]};\ F_N\epsilon[1.2061, 1.1902]$$

The proposed T-test can be applied under the assumption when two populations have equal variances. The equality of variances can be tested using the F-test first. Let $\alpha$=0.05, and confidence level (1- $\alpha$) is 0.95, and degree of freedom for sample 1 and sample 2 is 23. At these values, the critical value for F-test is 1.7158. As the calculated values of the statistic $F_N$ is less than 1.7158. It is concluded that the two population variances are equal. Therefore, the rock engineer can move ahead to use the neutrosophic T-test for testing the means of two populations. The neutrosophic-pooled variance for the data is given by

$$s_{pN} = \sqrt{\frac{23[1.1807, 2.0664] + 23[0.9789, 1.7360]}{46}};\ s_{pN}\epsilon[1.0391, 1.3788]$$

The neutrosophic t-test statistic is calculated as follows

$$t_N = \frac{(\bar{z}_{N1} - \bar{z}_{N2}) - d_{0N}}{s_{pN}\sqrt{\frac{1}{n_{N1}} + \frac{1}{n_{N2}}}} = \frac{[0.1687 - 1.0981] - [0,0]}{[1.0391, 1.3788]\sqrt{\frac{1}{24} + \frac{1}{24}}};\ t_N\epsilon[0.5623, 2.7587]$$

where $d_{0N} = [0,0]$ denote the difference between means of neutrosophic population.

The critical value for the t-test is 2.011. It is noted that the critical value is between the indeterminacy intervals of $t_N\epsilon[0.5623, 2.7587]$. Therefore, the decision about the acceptance of the null hypothesis is made using [22] approach. According to [22], the null hypothesis is rejected if min $t > table\ value$. Based on this criterion, the null hypothesis of equal means is not rejected. Therefore, it is concluded that both lengths have the same mean and the same variances.

## 4 Comparative Study

The advantages of the proposed neutrosophic F-test and neutrosophic t-test over the existing F-test and t-test under the classical statistics in terms of measures of indeterminacy are discussed. The latter is used when all the observations in the data are determined. The existing tests are special cases of the proposed tests. The proposed tests reduce to existing tests under classical statistics when $I_L = 0$. The proposed tests are compared with the existing tests for the rock roughness data given in Section 3. The neutrosophic numbers of the proposed F-test and t-test can be written as $F_N = 1.902 + 1.2061I_N$; $I_N \epsilon [0,0.01318]$ and $t_N = 0.5623 + 2.7587I_N$;$I_N \epsilon [0,0.7961]$. The values of statistic $F_N \epsilon [F_L, F_U]$ and $t_N \epsilon$ $[t_L, t_U]$ reduce to F-test and t-test under classical statistics when $I_L = 0$. The values 1.902 and 0.5623 in these neutrosophic forms present the values of the F-test and t-test under classical statistics, respectively. The values $1.2061I_N$ and $2.7587I_N$ present the indeterminate parts of the proposed F-test and t-test, respectively.

A brief summary of the proposed and the existing test is shown in Table 2.

**Table 2: The summary of tests**

| Proposed Tests | Proposed Test Statistic | Existing Tests | Exiting Test Statistic |
|---|---|---|---|
| F-test | $F_N \epsilon [1.2061,1.1902]$;$I_N \epsilon [0,0.01318]$ | F-test | 1.1902 |
| t-test | $t_N \epsilon [0.5623,2.7587]$; $I_N \epsilon [0,0.7961]$ | t-test | 0.5623 |

From Table 2, it is noted that the proposed tests provide the test statistic values in indeterminacy intervals when uncertain observations are present in the data. On the other hand, the existing tests provide the determined values of the test statistic. In addition, the proposed tests give information about the measure of indeterminacy while the existing tests do not provide such information. For example, the proposed F-test provides the measure of indeterminacy that is 0.0131 and the proposed t-test gives the measure of indeterminacy which is 0.7961. Based on the information, the proposed F-test can be interpreted as the probability that $H_{0N}$:$\sigma_{N1}^2 = \sigma_{N2}^2$ is accepted is 0.95, the probability of rejecting $H_{0N}$:$\sigma_{N1}^2 = \sigma_{N2}^2$ is 0.05 and the probability of uncertainty about the null hypothesis is 0.0131. Similarly, the proposed t-test can be interpreted as the probability that $H_{0N}$: $\mu_N = \mu_{0N}$ is accepted is 0.95, the probability of rejecting $H_{0N}$: $\mu_N = \mu_{0N}$ is 0.05 and the probability of uncertainty about the null hypothesis is 0.7961. From this study, it is concluded that the proposed tests are more informative and efficient than the existing tests. By comparing these tests, it can conclude that the proposed tests are appropriate to be applied under uncertain settings. Therefore, the proposed tests are quite effective to test $H_{0N}$:$\sigma_{N1}^2 = \sigma_{N2}^2$ and $H_{0N}$: $\mu_N = \mu_{0N}$ in the presence of uncertainty.

## 5 Concluding Remarks

The F-test and t-test under the neutrosophic statistical interval method were presented in this paper. The design and decision criteria of both tests are developed and discussed with the help of a real example. The proposed tests under neutrosophic statistics are the generalization of the tests under classical tests. From the comparative study, it is concluded that the proposed tests expressed the results in indeterminacy interval rather than the exact value when data is imprecise, in interval and indeterminate. The proposed tests can be applied in a variety of fields where indeterminacy and uncertainty in the data are expected. The proposed test has some limitations that it can be applied when two populations have equal variances and the samples are drawn from the neutrosophic normal distribution. The proposed test for neutrosophic non-normal distributions can be extended for future research. The proposed test when variances of two populations are not equal can be developed as future research. The proposed tests for multivariate neutrosophic data can be considered as future research.

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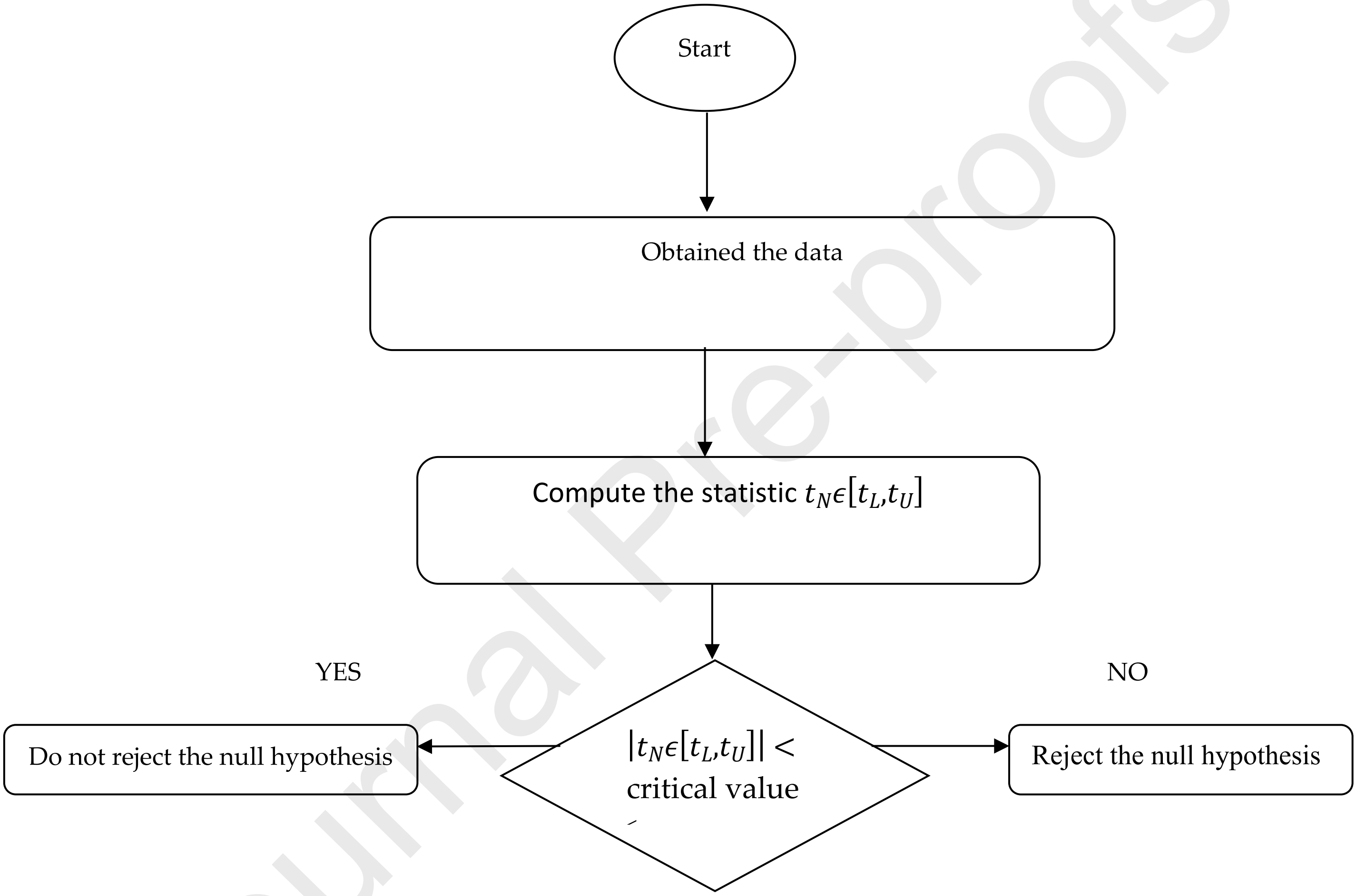
Start
Obtained the data
Compute the statistic $t_N \epsilon [t_L, t_U]$
YES
NO
Do not reject the null hypothesis
$|t_N \epsilon [t_L, t_U]| <$ critical value
Reject the null hypothesis

**Figure 1: The operational process of the proposed t-test**

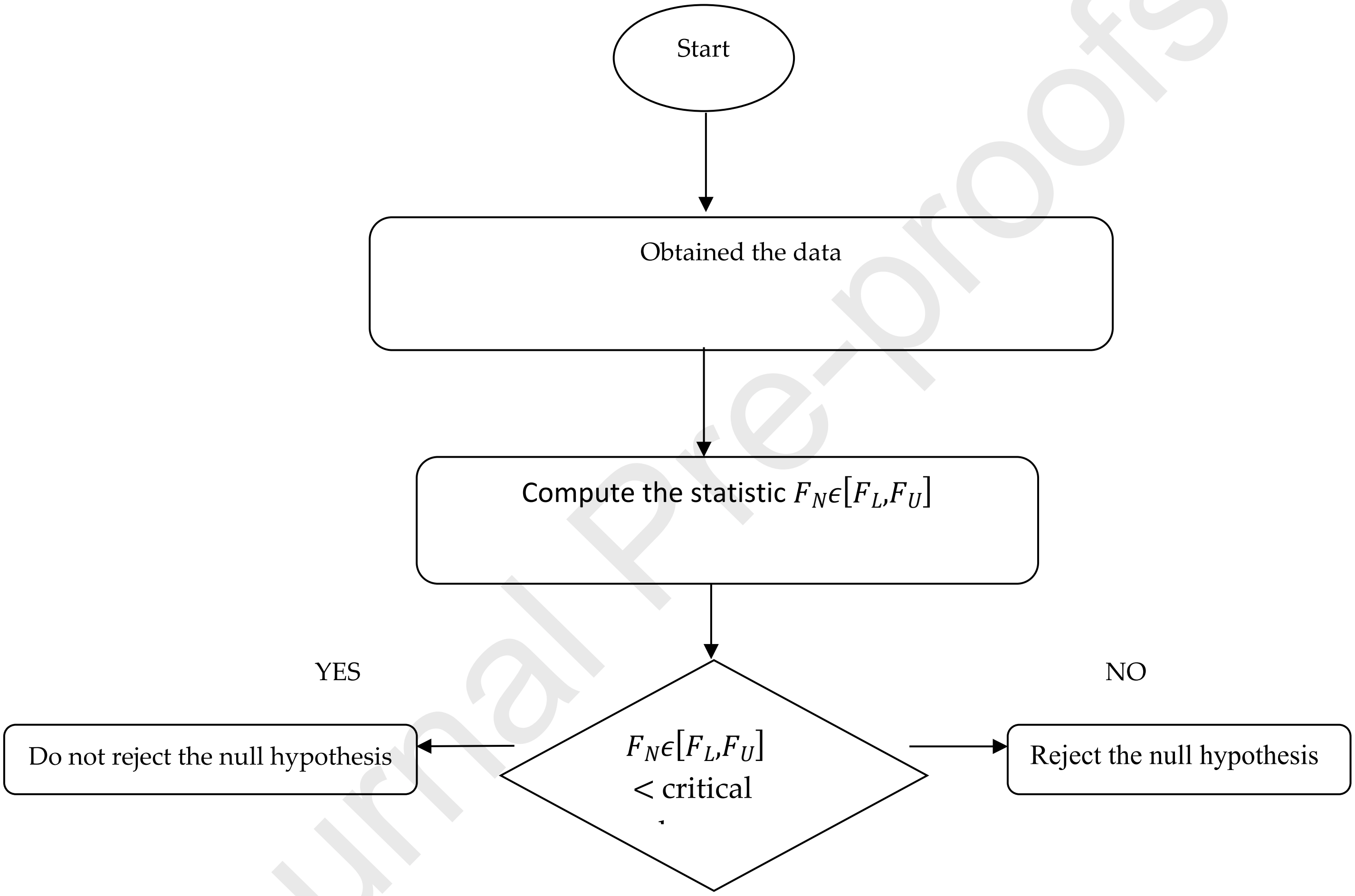
Start
Obtained the data
Compute the statistic $F_N \epsilon [F_L, F_U]$
YES
NO
Do not reject the null hypothesis
$F_N \epsilon [F_L, F_U]$ < critical
Reject the null hypothesis

**Figure 2: The operational process of the proposed F-test**